ELSEVIER

Hoghton's camera and telescope

Addolorata Marasco, Antonio Romano *

Department of Mathematics and Applications, University of Naples Federico II, Via Cintia, 80126 Naples, Italy

Abstract

The third-order aberration formulae we have proved in a previous paper, starting from Fermat's principle and from the idea of stigmatic paths, are here applied to analyze and to project Houghton's cameras and Houghton–Cassegrain's telescopes. The final values of the radii of the two-lens corrector are obtained taking into account also the thickness of lenses and the fifth-order aberrations.
© 2006 Elsevier Ltd. All rights reserved.

Keywords: Geometrical optics; Aberration formulae; Houghton's cameras; Houghton–Cassegrain's telescopes

1. Introduction

In [1] we proposed a new approach to geometrical optics based both on Fermat's principle and on the introduction of suitable optical paths: the *stigmatic paths*. Although these paths are not rays, they have a known behavior and can be used instead of the effective rays to evaluate the optical path lengths up to fourth-order terms in the coordinates of the rays. All the third-order aberration theory of an optical system with a symmetry of revolution around an axis is easily derived without resorting to the whole Hamiltonian formalism and Seidel's variables (see [4–6]). The formulae which describe the third-order aberrations are different from Seidel's ones but give the same numerical results. In this paper we apply our formulae to particular astronomical optical systems: Houghton's cameras and telescopes.

Let us consider the optical system formed by a spherical mirror S and a circular stop whose center is located at the center of curvature of S. If $-R$ denotes the curvature radius of S, then it is well known that S forms an image of an object at infinity on the spherical surface σ of radius $f = -R/2$ which crosses the optical axis a at a point of abscissa $-f$. This image, owing to the symmetry of this combination with respect to the center A of S, is not affected by off-axis aberrations (coma, astigmatism and distortion) but is affected by a substantial spherical aberration. In 1920s the optician Schmidt introduced in the plane of the stop a corrector C, i.e., a thin plate having a planar face and the other one with a fourth-order profile to balance the spherical aberration of the primary mirror S. The resulting astronomical camera supplies a very sharp image on the sphere σ

---

* Corresponding author.
E-mail addresses: marasco@unina.it (A. Marasco), antroman@unina.it (A. Romano).

but the length of the tube containing C and S becomes too long when the focal f of the primary mirror increases. Moreover, in order to obtain the required fourth-order profile a very difficult and expensive manufacturing is necessary. For these reasons, opticians tired to design a camera in which the Schmidt aspheric corrector was replaced by a corrector that could be more easily manufactured. In this paper we are interested in the combination proposed in 1944 by Houghton [3].[1]

Houghton's corrector is an afocal combination C of two or three lenses which are made with the same kind of glass. This corrector introduces a spherical aberration compensating for that of the mirror S. Buchroeder [7] introduced at the center of curvature of the primary mirror S a suitable afocal set of three lenses. An exhaustive analysis of the two-lens correctors was carried out by Sigler [8] and, more recently, by Blakley and Riccardi [9] both for cameras and Cassegrain telescopes (see also [10]). This analysis leads to the following results: it is possible to compensate the spherical aberration and coma almost for any position of the corrector with respect to S. However, in order to control the astigmatism, it is necessary to use a conic mirror S.

All the above results are derived assuming that:

- the thickness of the lenses can be neglected;
- the conditions to apply the third-order aberration theory are satisfied.

These assumptions lead to data that cannot be accepted as final values of the radii of the lenses composing the corrector of the camera or of the telescope. However, these data represent a good starting point to apply one of the optimization methods contained in the professional programs for the analysis of an optical system. It is important to remark that these methods, which are based on the minimization of a suitable error function, require many attempts and frequently do not supply a satisfactory result.

In this paper, we again analyze cameras and Cassegrain telescopes with a Houghton's two-lens corrector starting from the third-order aberration formulae derived in [1]. We reach the following results:

1. The dependence of the radii of the lenses forming the corrector, which eliminate spherical aberration and coma, is determined as function of the corrector position, neglecting the thickness of lenses. In such a way it is possible to locate the corrector in the positions corresponding to largest radii in order to reduce the higher-order aberrations and make easier the manufacturing. This analysis is carried out also for correctors having different radii. In particular, it is shown that this choice leads to larger radii of the surfaces of the corrector with respect to the case of correctors with the same bendings of the two lenses.
2. The equations to have an afocal corrector with given spherical aberration and coma when the lenses have a given thickness are determined by resorting Mathematica. Moreover, still by the software Mathematica, these equations are solved by Newton's method using the approximate values obtained in the item 1.
3. These optical combinations are analyzed with the professional optical program OSLO to evaluate the fifth-order spherical aberration and coma. In such a way it is possible to apply the procedure of item 2 imposing that the corrector has third-order spherical aberration and coma to balance the higher-order ones.

It is worthwhile to remark that the final project does not need any further correction.

In Section 2 we recall the notations and formulae of paper [1]. In Sections 3–5 the Houghton cameras are analyzed whereas Section 6 is devoted to the Hougthon–Cassegrain telescopes. Sections 3 and 4 conclude with spot diagrams referring to cameras in which the primary mirror has a focal ratio $f/3.5$ and $f/3$, respectively. At the end of Section 5 a spot diagram of a camera using a conic mirror with a focal ratio $f/2.5$ is showed. Finally, Section 6 ends with a spot diagram referring to a Hougthon–Cassegrain telescope with a $f/2.5$ primary mirror. In all the examined cases the classical approximations do not give satisfactory results. The proposed projects employ the common BK7 glass.

---

[1] In the paper [2] we have analyzed Schmidt's cameras.

2. Third-order monochromatic aberration formulae

In this section we recall the notations we used in [1] as well as the third-order monochromatic aberration formulae we derived in that paper.

Let S be an optical system composed by n surfaces of revolution $S_1, \ldots, S_n$, having a rotational symmetry around a common axis a which is called the *optical axis*. These surfaces have curvature radii $R_1, \ldots, R_n$ at the vertices $O_1, \ldots, O_n$, where they intersect a (see Fig. 1). Moreover, let t_i be the distance of S_{i+1} from S_i, evaluated along a, and N_i, $N_i' \equiv N_{i+1}$ the refractive indices of the media before and after S_i, respectively. With $O_i x_i y_i z_i$, $i = 1, \ldots, n$, we denote Carthesian frames of reference having their origins at the vertices O_i and the Oz_i-axes coinciding with a. Finally, all the axes $O_i x_i$, $i = 2, \ldots, n$, are chosen parallel to $O_1 x_1$.

Let π_1 be an object plane, orthogonal to a, and Ω a circular region of π_1, with its center at the intersection point between a and π_1 (see Fig. 1). Usually, the purpose of the optical project is to determine which optical and geometrical characteristics of S guarantee that the whole circle Ω is transformed by S into a circle Ω', whose center coincides with $a \cap \pi_n'$, where π_n' is a prefixed image plane. In other words, it is required that the correspondence between the coordinates (x_1, y_1) of any object point belonging to Ω and the coordinates (x_n', y_n') of the corresponding image point of Ω' are given by the relations:

$$x_n' = M_{1,n} x_1, \quad y_n' = M_{1,n} y_1, \tag{1}$$

where the *constant* $M_{1,n}$ is said the *magnification factor* of S relative to the planes π_1 and π_n'. In particular, Eqs. (1) imply that *the image preserves the form of the object*.

It is well known that, for small values of the object coordinates and rays which form small angles with the optical axis (Gaussian approximation), the formulae

$$\frac{N_i'}{R_i} - \frac{N_i'}{z_i'} = \frac{N_i}{R_i} - \frac{N_i}{z_i}, \tag{2}$$

$$M_{i,i+1} = \frac{N_i z_i'}{N_i' z_i}, \tag{3}$$

together with the relations

$$z_{i+1} = z_i' - t_i, \quad N_{i+1} = N_i', \quad y_{i+1} = y_i', \tag{4}$$

allow us to evaluate the pair of conjugate planes for any surface S_i and the relative magnification between these planes so that the final position of the image plane π_n' and the total magnification

$$M_{1,n} = M_{1,2}, \ldots, M_{n-1,n} \tag{5}$$

are determined.

Finally, supposing that the optical system includes an aperture stop π_a, we denote by S_{en} the part of S before π_a and by S_{ex} the part which follows π_a. Then, the conjugate plane π_e of π_a with respect to S_{en} is called the *entrance pupil* of S, whereas the conjugate plane π_e' of π_a with respect to S_{ex} is the *exit pupil* of S. We denote by w the distance of π_e from the vertex O_1 of the first surface S_1 of S.

It is evident that the behavior of any ray starting from the object point $(x_1, y_1) \in \pi_1$ and crossing the entrance pupil at the point (x_e, y_e) is completely determined. On the other hand, owing to the symmetry of revolution of S around the optical axis a, it is sufficient to consider only object points belonging to the axis
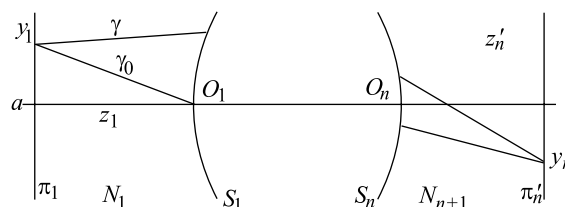

Fig. 1. Optical system.

O_1y_1. For these reasons, the numbers (y_1, x_e, y_e) will be called the *coordinates* of the ray. From the refraction and reflection laws we derive that the ray (y_1, x_e, y_e) intercepts the image plane π'_n at a point

$$x'_n = x'_n(y_1, x_e, y_e), \quad y'_n = y'_n(y_1, x_e, y_e). \tag{6}$$

The functions

$$\epsilon_x = x'_n(y_1, x_e, y_e), \quad \epsilon_y = y'_n(y_1, x_e, y_e) - M_{1,n}y_1 \tag{7}$$

are the *aberration functions* of S. Owing to the symmetry of revolution of S, their power expansion includes only odd power terms of degree 3, 5, etc. If only the first terms are taken into account, then we consider the *primary aberrations* or the *third-order aberrations*. We note that the variable y_1 can be substituted by the angle θ that the ray forms with the optical axis. This angular variable is necessary when the object is at infinity.

In [1] we have proved that the third-order aberration functions have the following form:

$$\epsilon_x = \frac{z'_n - w'_n}{N'_n M_{e,1,n}} (4D_1(x_e^2 + y_e^2)x_e + 2D_2 x_e y_e \theta + 2D_3 y_e \theta^2), \tag{8}$$

$$\epsilon_y = \frac{z'_n - w'_n}{N'_n M_{e,1,n}} (4D_1(x_e^2 + y_e^2)y_e + D_2(x_e^2 + 3y_e^2)\theta + 2D_4 y_e \theta^2 + D_5 \theta^3), \tag{9}$$

where w'_n denotes the distance of the exit pupil from the last surface S_n and $M_{e,1,n}$ is the magnification factor relative to the pair of conjugate planes π_e, π'_e. It remains to define the expressions of the coefficients $D_1, \ldots, D_5$.

To this end, we first define the quantities relative to any surface S_i

$$C_{i,1} = -\frac{N_i^2}{8} \left(\frac{1}{N'_i z'_i} - \frac{1}{N_i z_i}\right) \left(\frac{1}{z_i} - \frac{1}{R_i}\right)^2 + \frac{N'_i - N_i}{8R_i^3} - (N'_i - N_i)a_{i,4}, \tag{10}$$

$$C_{i,2} = \frac{N_i^2}{2} \left(\frac{1}{N'_i z'_i} - \frac{1}{N_i z_i}\right) \left(\frac{1}{z_i} - \frac{1}{R_i}\right), \tag{11}$$

$$C_{i,3} = \frac{N_i}{4} \left(\frac{N'^2_i - N_i^2}{N'^2_i}\right) \left(\frac{1}{z_i} - \frac{1}{R_i}\right), \tag{12}$$

$$C_{i,4} = C_{i,3} - \frac{N_i^2}{2} \left(\frac{1}{N'_i z'_i} - \frac{1}{N_i z_i}\right), \tag{13}$$

$$C_{i,5} = -\frac{N_i}{2z_i^3} \frac{N'^2_i - N_i^2}{N'^2_i}, \tag{14}$$

where $a_{i,4}$ denotes the aspheric constant which appears in the approximate equation of the surface S_i

$$Z_i = \frac{1}{2R_i}(X_i^2 + Y_i^2) + a_{i,4}(X_i^2 + Y_i^2)^2. \tag{15}$$

In particular, when S_i is obtained rotating about a a conic with a conic constant K, it is

$$a_{i,4} = \frac{1 + K}{8R_i^3}. \tag{16}$$

Then, we introduce the quantities

$$c_i = \frac{z_i}{z_i - w_i}, \quad d_i = \frac{w_i}{z_i - w_i}, \tag{17}$$

as well as the coefficients

$$C^*_{i,1} = c_i^4 C_{i,1}, \tag{18}$$

$$C^*_{i,2} = c_i^2(C_{i,2} - 4z_i d_i C_{i,1}), \tag{19}$$

$$C^*_{i,3} = C_{i,3} - z_i d_i C_{i,2} + 2z_i^2 d_i^2 C_{i,1}, \tag{20}$$

$$C_{i,4}^* = C_{i,4} - 3z_i d_i C_{i,2} + 6z_i^2 d_i^2 C_{i,1}, \tag{21}$$

$$C_{i,5}^* = \frac{1}{c_i^2}(C_{i,5} - 2z_i d_i C_{i,4} + 3z_i^2 d_i^2 C_{i,2} - 4z_i^3 d_i^3 C_{i,1}). \tag{22}$$

Finally, we have that

$$D_1 = C_{1,1}^* + \sum_{i=2}^{n} M_{e,1,i-1}^4 C_{i,1}^*, \tag{23}$$

$$D_2 = C_{1,2}^* + N_1 \sum_{i=2}^{n} \frac{1}{N_i} M_{e,1,i-1}^2 C_{i,2}^*, \tag{24}$$

$$D_3 = C_{1,3}^* + N_1^2 \sum_{i=2}^{n} \frac{1}{N_i^2} C_{i,3}^*, \tag{25}$$

$$D_4 = C_{1,4}^* + N_1^2 \sum_{i=2}^{n} \frac{1}{N_i^2} C_{i,4}^*, \tag{26}$$

$$D_5 = C_{1,5}^* + N_1^3 \sum_{i=2}^{n} \frac{1}{N_i^3 M_{e,1,i-1}^2} C_{i,5}^*. \tag{27}$$

Let us suppose that the distances t_i, $i = 1,\ldots,n-2$ among the first $n-1$ surfaces of the optical system S can be neglected. If the aperture stop is located on the first surface S_1, then the first $n-1$ surfaces do not modify the entrance and the exit pupils which coincide with the aperture stop. Moreover, if the system $S_1,\ldots,S_{n-1}$ is afocal, i.e., its total focal is infinity, and the object is at infinity, then this subsystem does not modify the position of the object. In conclusion, the last surface S_n sees the object at infinity and the aperture stop located on the surface S_1. In other words, it is possible to evaluate the aberration coefficients of S_n without considering the presence of the previous surfaces.

Since in the sequel we consider optical systems satisfying the above hypotheses, we now evaluate the aberration coefficients of an optical system formed by a single concave conic mirror S with curvature radius $R = -2f$ and an aperture stop located in front of S at a distance δf. We remark that $f = -R/2$ denotes the *absolute* value of the focal length of S. Since the object is at infinity and $N = -N' = 1$, from (2) we derive the distance of the image from the mirror

$$z' = -f. \tag{28}$$

In order to find the distance w' of the exit pupil from S we note that $w = \delta f$ so that from (3), (4) and (17) we obtain

$$w' = \frac{f\delta}{1-\delta}. \tag{29}$$

$$M_e = \frac{1}{1-\delta}, \quad c = 1, \quad d = 0. \tag{30}$$

Moreover, the coefficients (10)–(13) become

$$C_1 = -\frac{K+1}{32f^3}, \tag{31}$$

$$C_2 = \frac{1}{4f^2}, \tag{32}$$

$$C_3 = 0, \tag{33}$$

$$C_4 = -\frac{1}{2f}, \tag{34}$$

whereas the relations (18)–(21) assume the form

$$C_1^* = -\frac{K+1}{32f^3}, \tag{35}$$

$$C_2^* = \frac{1}{8f^2}(2 - (K+1)\delta), \tag{36}$$

$$C_3^* = \frac{\delta}{16f}(4 - (K+1)\delta), \tag{37}$$

$$C_4^* = -\frac{1}{16f}(8 - 12\delta + 3(K+1)\delta^2). \tag{38}$$

Since we have just one surface, the coefficients of total spherical aberration and coma, i.e., $D_{S,1}$ and $D_{S,2}$ given by (23) and (24), coincide with (35) and (36)

$$32f^3 D_{S,1} = -(K+1), \tag{39}$$
$$8f^2 D_{S,2} = 2 - (K+1)\delta, \tag{40}$$

whereas the total coefficients of astigmatism and field curvature are

$$8f(D_{S,4} - D_{S,3}) = 8f(C_4^* - C_3^*) = -(4 - 4\delta + (K+1)\delta^2), \tag{41}$$

$$D_{S,4} - 3D_{S,3} = C_4^* - 3C_3^* = -\frac{1}{2f}. \tag{42}$$

Moreover, the Petzval curvature radius is

$$R_p = -f, \tag{43}$$

We conclude this section noting that the calculations necessary to find the expressions of the aberration coefficients of a complex system are very heavy. For this reason we have implemented a program by Mathematica which evaluates *analytically* the aberration coefficients of the cameras or Cassegrain telescopes considered in the next sections.

3. Houghton's camera: first form

The optical scheme we consider in this section is represented in Fig. 2. It is formed by a concave mirror S with an aperture stop A located in front of it at a distance δf, where $f = -R/2$ is the focal length of S and $R(<0)$ its radius of curvature. At the plane of A there is a corrector H formed by two spherical lenses, made by the same glass, whose curvature radii R_i, $i = 1, \ldots, 4$, satisfy the conditions

$$R_1 = -R_3, \quad R_2 = -R_4. \tag{44}$$

When the thickness of these lenses goes to zero and (44) are verified, an afocal system is obtained *for any wavelength* since the focal length of two adjacent thin lenses is given by the relation
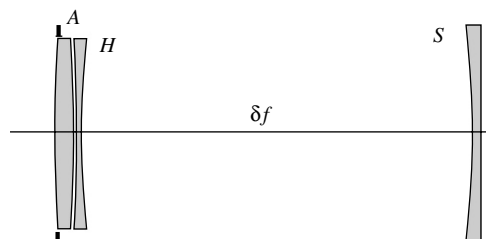
$$f = \frac{1}{(c_1 - c_2 + c_3 - c_4)(N-1)}, \tag{45}$$

Fig. 2. Houghton's camera.

where $c_i = 1/R_i$, $i = 1,\ldots,4$ denote the curvatures of the corrector surfaces and N is the refractive index of the glass.

The expressions of the spherical aberration, coma, and astigmatism coefficients of S are obtained by (39)–(41) for $a_4 = K = 0$

$$D_{S,1} = -\frac{1}{32f^3},$$ (46)

$$D_{S,2} = \frac{1}{8f^2}(2 - \delta),$$ (47)

$$D_{S,4} - D_{S,3} = -\frac{1}{8f}(2 - \delta)^2.$$ (48)

If the thickness of the lens forming Houghton's corrector is supposed to be zero the following aberration coefficients are obtained:

$$D_{H,1} = (c_1 - c_2)^2(c_1 + c_2)\frac{(N+1)(N-1)^2}{4N},$$ (49)

$$D_{H,2} = -(c_1^2 - c_2^2)\frac{(N^2 - 1)}{2N},$$ (50)

$$D_{H,4} = D_{H,3}.$$ (51)

The system is aplanatic when the curvature radii verify, for assigned $D_{S,1}$, $D_{S,2}$, the following equations:

$$D_{H,1} + D_{S,1} = 0, \quad D_{H,2} + D_{S,2} = 0.$$ (52)

When the expressions (49) and (50) are inserted into (52), the sixth-degree system is obtained:

$$(c_1 - c_2)^2(c_1 + c_2)\frac{(N+1)(N-1)^2}{4N} = -D_{S,1},$$ (53)

$$(c_1^2 - c_2^2)\frac{(N^2 - 1)}{2N} = D_{S,2},$$ (54)

whose solutions supply the curvatures of the two lenses which eliminate spherical aberration and coma. It is very important to remark that this system is equivalent to a linear one, so that the aforesaid curvatures are uniquely determined. In fact, eliminating $c_1^2 - c_2^2$ from (54) and (53) one has

$$c_1 - c_2 = -\frac{2D_{S,1}}{(N-1)D_{S,2}}.$$ (55)

Taking into account this formula, (53) and (54) become

$$c_1 - c_2 = -\frac{2D_{S,1}}{(N-1)D_{S,2}},$$ (56)

$$c_1 + c_2 = -\frac{ND_{S,2}^2}{(N+1)D_{S,1}},$$ (57)

whose unique solution is

$$c_1 = -\frac{2(N+1)D_{S,1}^2 + N(N-1)D_{S,2}^3}{2(N^2 - 1)D_{S,1}D_{S,2}},$$ (58)

$$c_2 = \frac{2(N+1)D_{S,1}^2 - N(N-1)D_{S,2}^3}{2(N^2 - 1)D_{S,1}D_{S,2}}.$$ (59)

Finally, using (46) and (47) and recalling that $R_i = 1/c_i$, the radii of the doublet are obtained

$$R_1 = f\frac{4(N^2 - 1)(2 - \delta)}{(N + 1) + N(N - 1)(2 - \delta)^3}, \tag{60}$$

$$R_2 = -f\frac{4(N^2 - 1)(2 - \delta)}{(N + 1) - N(N - 1)(2 - \delta)^3}. \tag{61}$$

Moreover, the astigmatism is absent when (see (48) and (51)):

$$(D_{H,4} + D_{S,4}) - (D_{H,3} + D_{S,3}) = D_{S,4} - D_{S,3} = -\frac{1}{8f}(2 - \delta)^2 = 0, \tag{62}$$

i.e., when the aperture stop A is situated at the curvature center of S ($\delta = 2$) and increases when A is closer to the mirror S. In other words, the astigmatism of the whole combination coincides with the astigmatism of the primary mirror, since the doublet does not contribute to it, at least when the thickness of the lenses constituting the doublet vanishes. However, we cannot eliminate the astigmatism by positioning the aperture stop at the curvature center of the mirror since, when $\delta \to 2$, owing to (60) and (61), the radii R_1 and R_2 go to zero. Fig. 3, in which $r_i = R_i/f$, shows graphically all these conclusions and suggest to choose $\delta \in [0.7, 1.2]$. We note that the continuous line refers to r_1, whereas the dashed line to r_2.

When $\delta \simeq 1.2$, the radii R_1 and R_2 are still sufficiently high and the astigmatism is acceptable for a photographic use and the tube is still shorter than in an equivalent Schmidt's Camera.

With the previous formulae, very good results are obtained if $\delta \simeq 0.7$ and the speed of S is slower than $f/4$. When the speed of S belongs to the interval $(f/4, f/3)$, the curvatures supplied by (60) and (61) are too strong so that higher-order aberrations appear and the astigmatism is not acceptable. Moreover, the thickness is not negligible and the corrector is not afocal since (45) is no more valid. However, the program **HCamera**, which we have written by Mathematica, uses the *exact* third-order formulae and solve them with Newton's method starting from the values of the radii given by (60) and (61). It is also possible to balance third and fifth-order spherical aberration and coma. Finally, the program modifies the value of the fourth radius in order have an afocal corrector also for finite thickness, reducing the chromatic aberrations.

Fig. 4 shows the spot diagram obtained with the professional program OSLO of a camera with a corrector having a diameter $D = 20$ cm, a principal mirror S with a focal ratio $f/3.5$ and a total length of 85 cm.

This camera satisfies the photographic criterion since the aberration spot has a diameter less than 0.025 mm on a film 24×36 mm.

4. Houghton's camera: second form

A way to reduce the curvature of the lenses, and consequently the higher-order aberrations, consists in adopting *four* different values of the radii of the lenses forming the doublet. Then, we

- impose that the value of c_4 makes afocal the doublet (see (45)):
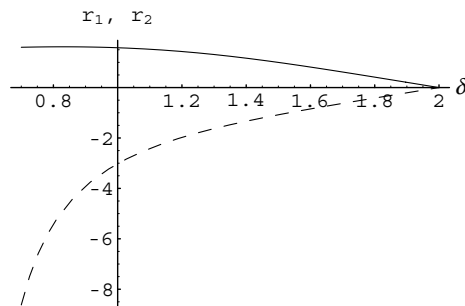
$$c_4 = c_1 - c_2 + c_3; \tag{63}$$

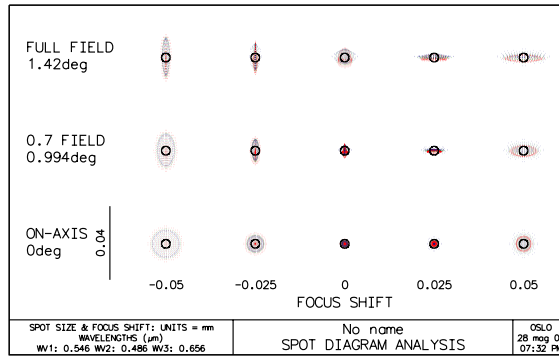Fig. 3. Radii versus distance from mirror.

Fig. 4. Spot diagram of a camera $f/3.5$.

- determine a convenient criterion to choose c_3;
- require that the remaining curvatures c_1 and c_2 make aplanatic the whole combination.

In order to satisfy the previous conditions, we use our program **HCamera** to show that the relations (53) and (54) are substituted by the following others:

$$(c_1 - c_2)(c_2 - c_3)\left(-\frac{1}{8}(3c_1 - 2c_2 + c_3)(N + 1) + \frac{1}{4N}(2c_1 - c_2 + c_3) + \frac{1}{4}(c_1 - c_2)N^2\right) = -D_{S,1}, \tag{64}$$

$$-(c_1 - c_2)(c_2 - c_3)\frac{N^2 - 1}{2N} = -D_{S,2}. \tag{65}$$

Moreover, also in this case, the doublet does not contribute to the astigmatism since

$$D_{H,4} - D_{H,3} = 0. \tag{66}$$

Again, this system, which looks to be of the sixth degree, is equivalent to a second-degree system. In fact, with the notations

$$x = \frac{c_1}{c_3} = \frac{R_3}{R_1}, \quad y = \frac{c_2}{c_3} = \frac{R_3}{R_2}, \tag{67}$$

it can be written as

$$(y - 1)(x - y)\left(-\frac{1}{8}(3x - 2y + 1)(N + 1) + \frac{2x - y + 1}{4N} + \frac{N^2}{4}(x - y)\right) = -D_{S,1}R_3^3, \tag{68}$$

$$(x - y)(y - 1)\frac{N^2 - 1}{2N} = D_{S,2}R_3^2. \tag{69}$$

Writing (69) in the form

$$(x - y)(y - 1) = \frac{2N}{N^2 - 1}D_{S,2}R_3^2 \tag{70}$$

and substituting this relation into (68), we obtain the second-degree system

$$-\frac{1}{8}(3x - 2y + 1) + \frac{2x - y + 1}{4N} + \frac{N^2}{4}(x - y) = -\frac{N^2 - 1}{2N}\frac{D_{S,1}}{D_{S,2}}R_3, \tag{71}$$

$$(x - y)(y - 1) = \frac{2N}{N^2 - 1}D_{S,2}R_3^2. \tag{72}$$

Taking into account (46) and (47), we finally have

$$a_1 x + a_2 y = d_1, \tag{73}$$
$$(x - y)(y - 1) = d_2. \tag{74}$$

where

$$a_1 = \frac{4 - 3N - 3N^2 + 2N^3}{8N}, \tag{75}$$

$$a_2 = \frac{-1 + N + N^2 - N^3}{4N}, \tag{76}$$

$$d_1 = -\frac{1}{8} - \frac{1}{4N} + \frac{N}{8} + \frac{(N^2 - 1)r_3}{8N(2 - \delta)}, \tag{77}$$

$$d_2 = \frac{N}{4(N^2 - 1)}(2 - \delta)r_3^2, \quad r_3 = \frac{R_3}{f}. \tag{78}$$

This system has the following two solutions:

$$r_1 = 2r_3 \frac{a_1(a_1 + a_2)}{-a_1(a_2 - 2d_1) + a_2(-a_2 + d_1 + \sqrt{\Delta})}, \tag{79}$$

$$r_2 = 2r_3 \frac{a_1 + a_2}{a_1 + a_2 + d_1 - \sqrt{\Delta}}, \tag{80}$$

or

$$r_1 = -2r_3 \frac{a_1(a_1 + a_2)}{a_1(a_2 - 2d_1) + a_2(a_2 - d_1 + \sqrt{\Delta})}, \tag{81}$$

$$r_2 = 2r_3 \frac{a_1 + a_2}{a_1 + a_2 + d_1 + \sqrt{\Delta}}, \tag{82}$$

which are real, provided that the discriminant

$$\Delta = (a_1 + a_2 + d_1)^2 - 4(a_1 + a_2)(d_1 + a_1 d_2) \tag{83}$$

is not negative.

Substituting (75)–(78) into (79)–(82), all the acceptable values of r_3 corresponding to a given δ can be determined. The subset of the plane (r_3, δ), for which Δ is not negative, is represented in Fig. 5.

By Mathematica it is possible to analyze the behavior of the radii r_1 and r_2 on varying r_3 in the set I_δ of the acceptable values corresponding to a given δ. Figs. 6 and 7 plot the variables r_1, r_2 and r_4 versus r_3 for $\delta = 0.8$. More precisely, the first figure refers to the solution (79) and (80) and the second one to (81) and (82). The chosen interval for δ starts from the lower bound of I_δ.

In all the figures of this section the continuous line refers to r_1, the line with longer dotting to r_2 and the line with shorter dotting to r_4. Finally, Figs. 8 and 9 correspond to $\delta = 1.1$.
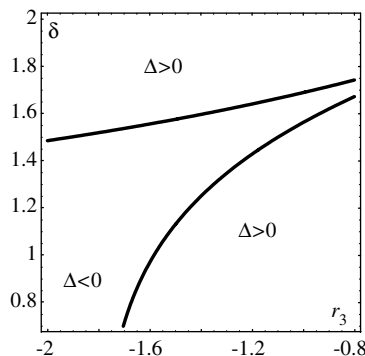

Fig. 5. δ versus r_3.

An analysis of the plots leads us to the following conclusions:

- the highest and, therefore, the most convenient values of r_1, r_2, r_3 and r_4 are obtained for $\delta \in [0.7, 1]$;
- for an assigned δ in this interval, the solution (79) and (80) and r_3, r_4 assume the highest values at the lower bound of I_δ, whereas the solution (81), (82) and r_3, r_4 have to be evaluated at 0.8–0.9 of the lower bound. For instance, for $\delta = 0.8$, from Fig. 5 we derive $I_\delta = [-1.65, -0.8]$, so that the solution (79) and (80) has to be evaluated at $r_3 = -1.65$, whereas (81) and (82) at $r_3 = -1.5$ (see Figs. 6 and 7, which refers to solutions (79)–(82), respectively);
- in any case, the second solution leads to greater values of the radii.

The program **HCamera** allows us to project a Houghton's camera with different radii, taking into account the thickness of lenses. Moreover, it supplies the possibility to require residual amounts of third-order spherical aberration and coma to balance the corresponding fifth-order aberrations. Finally, the radii obtained in
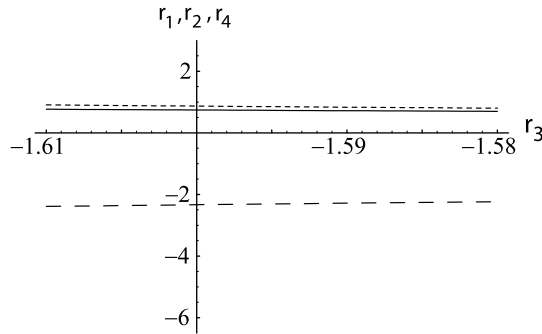

Fig. 6. Behaviour of the solution (79) and (80) for $\delta = 0.8$.
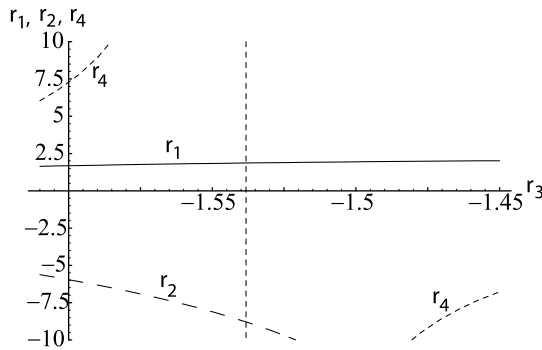

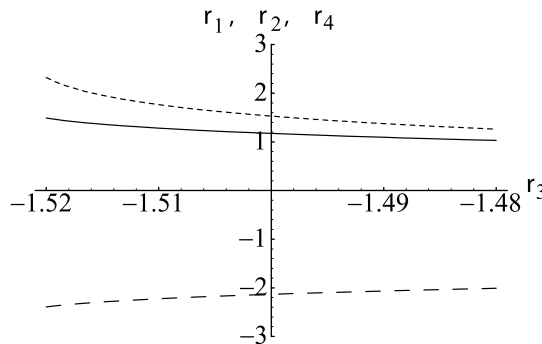Fig. 7. Behaviour of the solution (81) and (82) for $\delta = 0.8$.

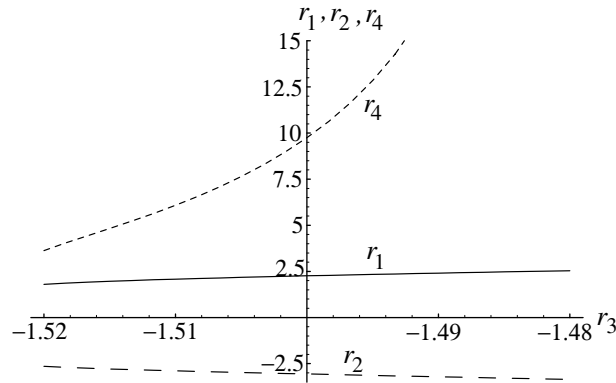Fig. 8. Behaviour of the solution (79) and (80) for $\delta = 1.1$.

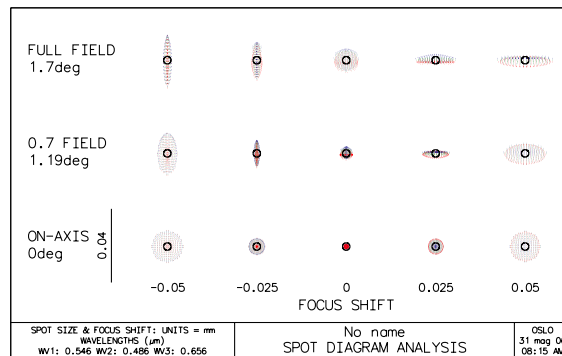Fig. 9. Behaviour of the solution (81) and (82) for $\delta = 1.1$.

Fig. 10. Spot diagram.

output make afocal the corrector for a not vanishing thickness of the lenses. Good results are obtained for mirror whose speed is slower or equal to $f/3$. Fig. 10 shows the spot diagram of a camera with a corrector with a diameter of 20 cm, a mirror $f/3$ and a total length equal to 75 cm.

5. Houghton's camera: third form

If we accept the idea to introduce aspherical surfaces, it becomes possible to eliminate astigmatism by adopting a conic mirror S. It has already been noted that the corrector does not introduce astigmatism when it is afocal and has no thickness. Consequently, the total astigmatism of the whole combination coincides with that one of the primary mirror. Taking into account (41), we reach the conclusion that *it is possible to eliminate this aberration for any position of the aperture stop*, that is for any $\delta > 0$, provided that the conic constant of S results

$$K = -\frac{(\delta - 2)^2}{\delta^2}. \tag{84}$$

For this value of the conic constant of S, the spherical aberration and coma coefficients (39) and (40) become

$$D_{S,1} = \frac{1 - \delta}{8f^3\delta^2}, \quad D_{S,2} = \frac{\delta - 2}{4f^2\delta}. \tag{85}$$

Finally, if these expressions are inserted into (58) and (59), the following formulae for the radii of a first-form doublet are derived:

$$R_1 = \frac{4\delta(N^2-1)(\delta-2)(\delta-1)}{-2(\delta-1)^2 + N^2\delta(\delta-2)^3 - N(2-12\delta+14\delta^2-6\delta^3+\delta^4)}, \tag{86}$$

$$R_2 = \frac{4\delta(N^2-1)(\delta-2)(\delta-1)}{2(\delta-1)^2 + N^2\delta(\delta-2)^3 - N(2+4\delta-10\delta^2+6\delta^3-\delta^4)}. \tag{87}$$

The behavior of the radii versus δ is shown in Fig. 11.

We conclude by noting that when $\delta \in [1.4, 2]$, the radii of the corrector surfaces become smaller and consequently higher-order aberrations appear. Convenient values of δ belong to the interval $[1.2, 1.4]$. Our program **HCamera** uses all the previous formulae to derive the curvature radii of Houghton's corrector as well as the conic constant of the mirror. Then, starting from these data and taking into account the thickness of the lenses, it evaluates the correct final values of the above quantities which balance the fifth-order aberrations. An example of the excellent performance of a camera of the third form, whose primary conic mirror S has a diameter $D = 200$ mm and a focal ratio $f/2.5$, is shown in Fig. 12.

All the results discussed in the above sections can be summarized as follows:

- if a conic mirror S is used, excellent Houghton's cameras can be obtained with speed up to $f/2.5$ and about 40% shorter than the corresponding Schmidt's cameras;
- with only spherical surfaces we can project good cameras with speed up to $f/3.5$, if the lens of the corrector have paired radii, and up to $f/3$, if the lens of the corrector have different radii. Moreover, these cameras are about 50% shorter than Schmidt's ones.
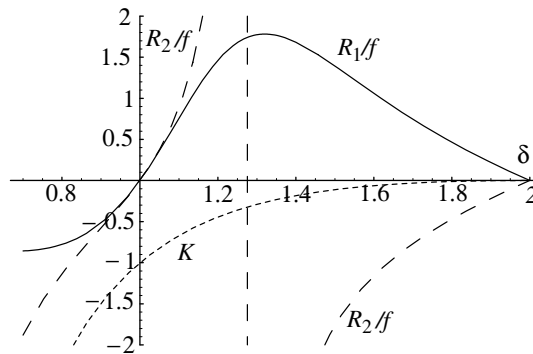

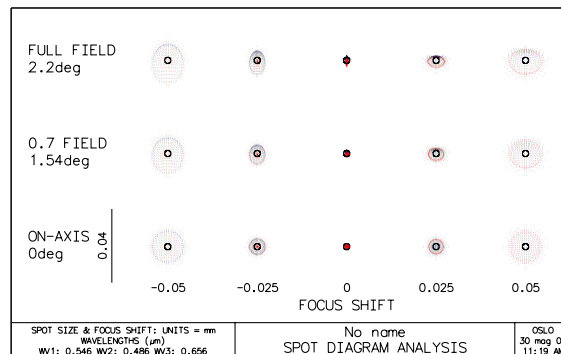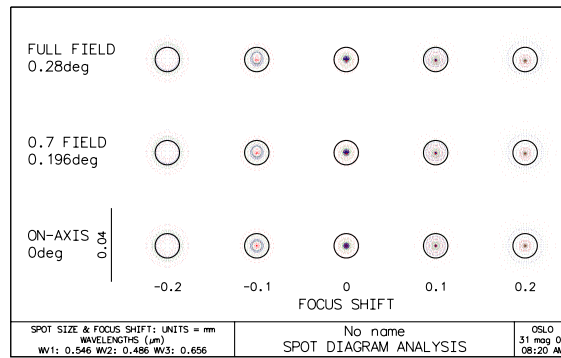Fig. 11. Behaviour of the solution (86) and (87).

Fig. 12. Spot diagram for a camera $f/2.5$.

Fig. 13. Spot diagram $f/2.5$.

6. Houghton–Cassegrain telescope

In this section, the Houghton–Cassegrain telescope is briefly considered. Essentially, neglecting the thickness of the lenses of corrector we can apply all the results described in the previous sections provided that, instead of the spherical aberration coefficient (46) and the coma coefficient (47) of a spherical mirror, we consider the corresponding coefficients of the Cassegrain combination (see, for instance [10]).

We only note that the elimination of the aberrations of this configuration is more simple than in the photographic camera for the following two reasons:

- owing to the longer focal of a telescope with respect to a camera, the field angle is smaller together with the off-axis aberrations;
- the secondary mirror partially compensate the aberrations of the primary one.

In conclusion, the corrector has to balance the residual aberrations of a Cassegrain configuration for smaller field angles. However, it is important to remark that the correction of aberrations has to be much more accurate, if the diffraction limit has to be reached.

We adopt the same method than for Houghton's camera. We have written by Mathematica the program **HCassegrain** which allows us to obtain the radii of curvature of the corrector taking into account both the finite thickness of the lenses and the balancing of third-order and fifth-order aberrations. Fig. 13 shows the spot diagram of a telescope based on this procedure. This telescope, whose equivalent focal is 200 cm, uses a primary mirror with a diameter of 20 cm and a speed $f/2.5$. This telescope is 40 cm length and has an obstruction factor of 31%. Noting that the circles in Fig. 13 represent the Airy disk, we can say that the telescope is diffraction limited.

References

[1] A. Marasco, A. Romano, Third order aberrations via Fermat's principle, Nuovo Cimento B 121 (1) (2006) 91–108.
[2] C. Bortone, A. Romano, Some remarks on the project of a Schmidt's camera, Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. 138 (2004) 27–36.
[3] J.L. Houghton, U.S. Patent 2,350,112. May 30, 1944.
[4] M. Born, E. Wolf, Principle of Optics, Pergamon, Oxford, 1980.
[5] R. Luneburg, Mathematical Theory of Optics, University of California Press, Berkeley, 1964.
[6] H. Crétien, Calcul des Combinations Optiques, Masson, 1980.
[7] R.A. Buchroeder, Telescope Making 8 (1980) 4.
[8] R.D. Sigler, Applied Optics 17 (10) (1978) 1519.
[9] R. Blakley, M. Riccardi, Applied Optics 42 (10) (2003) 1876.
[10] H. Rutten, M. van Venrooij, Telescope Optics, Willmann-Bell Inc., 1988.